Hints and Answers

Prologue

1. 50 **2.** 25 **3.** $T(j)=j+T(j-1)$ so $T(n)=n+(n-1)+\cdots+3+2+1=n(n+1)/2$. **4. a)** $S(j)=T(j)-d(j)$ **b)** $S(j)=\max[(i,j)\in E]\ S(i)+d(i)$. **5.** $T(j)=\max[h(e)=j]\{d(e)+T(t(e))\}$ where $d(e)$—duration of task, $h(e)$—vertex at head of e, $t(e)$—vertex at tail of e, $T(j)$—minimum time to get to vertex j and $h(e)=j$ means all tasks whose heads point to j. **6.** For n-vertex graph assume solution for $(n-1)$-vertex graph, assign 1 to some vertex with no arrow into it (why must there be such a vertex?), and then assign 2 to n to other vertices using $(n-1)$-vertex solution.

Chapter 0

Section 0.1

1. a) $\{\},\{x\},\{y\},\{z\},\{x,y\},\{x,z\},\{y,z\},\{x,y,z\}$ **2.** $P(\{a\})$ is $\{\emptyset,\{a\}\}$; $P(P(\{a\}))$ is $\{\emptyset,\{\emptyset\},\{\{a\}\},\{\emptyset,\{a\}\}\}$ **3. c)** all non-prime positive integers **4. a)** $\{x\,|\,100<x<200 \text{ and } 2\not|x\}$ **5. a)** $F\cap N^+$ **c)** $Q-Z$ **e)** $F\cap N^+\cap E$ **g)** $Z-E-N$ **i)** $Z-N$ **6. c)** $R-S=\{2,4\}$, $R-T=\{1,2\}$, $S-T=\{1,9\}$ **e)** $\{1,3,4,5,6,7,8\}$ **h)** $\{3\}$ **9.** No. (Why not?). **10. a)** $U-R$ is set of all things in U and not in R which is $\bar{r}R$ **c)** $R-(S\cap T)$ is the set of all things in R and not in both S and T and so is $(R-S)\cup(R-T)$ since only things in both S and T which are in R are not in $R-S$ or $R-T$. **11.** $|A\cup B|=|A|+|B|$ **13.** Nothing; give an example to show why. **16.** Least: m; Most: $m-n$ **18.** Build up a table with, for each value of p $(1\le p\le 10)$, the number of fractions p/q which have not already been counted. **20. b)** $(X\cap Y)\cap Z$ is the set of all things in both Z and in both X and Y, that is, the set of things in all three sets.

Section 0.2

1. a) $\{(x,y)\,|\,x,y\in R, x\le y\}$ **c)** $\{(x,y)\,|\,x,y\in Z, x\ne y\}$ **2. a)** reflexive, transitive **c)** symmetric **3. a)** $(1,5)$, $(2,6)$ are; $(1,6)$, $(2,5)$ are not. **b)** reflexive, symmetric and transitive **4. a)** $\in$ is not transitive; consider $a\in B$ and $B\in C$ where C is a set of sets. **b)** transitive **6. a)** $\{(x,y)\,|\,x<y\}$ **d)** R-reverse is the same as R. **e)** $\{(x,y)\,|\,x+1=y\}$ **7.** Yes for all three. **8. a)** $\{(s,t)\,|\,s<t\}$ **b)** itself **9. a)** child **b)** ancestor **10. a)** grandparent **c)** sibling (assuming you are your own sibling) **11.** 2: all even t, 36: all s which are factors of 36, namely 1, 2, 3, 4, 6, 9, 12, 18, 36 **13.** $\{(1,3),(2,4),(2,5),(4,6)\}$ **14.** Two pairs are (rhombus, quadrilateral), (square, parallelogram).

Section 0.3

1. b) 0, 1, 4, 9, 16, ... **2.** Injective **3.** If more than one student to a room, then not 1-to-1; if all rooms filled, then surjective. **5. a)** False **b)** True **6. a)** False **b)** False **7. a)** False **b)** False.

Section 0.4

3. a) 3 **6. a)** $0 \le x < 1$ **9. a)** Any integer **c)** All **e)** All **g)** Any integer or nonnegative real with fractional part $< 1/2$ or negative real with fractional part $> 1/2$ **i)** All multiples of n **10. a)** $\lfloor x+y \rfloor \ge \lfloor x \rfloor + \lfloor y \rfloor$ **c)** $\lfloor x-y \rfloor \ge \lfloor x \rfloor - \lceil y \rceil$ **e)** $n\lfloor x/n \rfloor \le \lfloor x \rfloor$, equal for all m, $nm \le x < nm = 1$ **11.** Let $w = x+y$ and use [10b]. **15. a)** $R'(x) = \lceil x + .5 \rceil - 1$ **17.** $\lfloor 10x + .5 \rfloor / 10$; rounds up if hundredths place is 5. **20. b)** 4 **c)** Need number of multiples of each power of 5; therefore $\lfloor n/5 \rfloor + \lfloor n/25 \rfloor + \lfloor n/125 \rfloor + \cdots$ **22.** Limit is 0. **23. a)** 15 **b)** 3; why are there no others? **25. b)** 7 **26. a)** 3 **28. a)** $\{x \mid x = 5n+2 \text{ and } n \in Z\}$ **c)** $\{x \mid x = 5n+4 \text{ and } n \in Z\}$ **30. a)** k even: $(2k)^2 = 4k^2$; k odd: $(2k+1)^2 = 4k(k+1)+1$. **31. a)** 3 **d)** 1 **g)** 10/3 **j)** 16 **32. a)** kL **c)** $-kL$ **33. c)** 10^{1000} **34. c)** 4 **36. a)** Domain: $x > 1$; Range: x real **38.** Fastest to slowest: $n^4/\log n$, n^3, n^2, $(\log n)^5$ **40.** Properties 1–3 hold but Property 4 does not; limit is $-\infty$ **41.** Product: $x^6 - 4x^5$ **43.** $x^4 - 1$ **46. a)** $f(x+1)/f(x) = 1 + (2x+1)/x^2$; $f(2x)/f(x) = 4$. **48. c)** 3014

Section 0.5

1. d) $\prod_{i=0}^{49}(2i+1)$ **2. d)** $x + \sum_{i=1}^{7}(i+1)x^{2i}$ **3. b)** 7451/110592 **4. b)** 2742 **5. a)** $1 - 1/5$ for $n = 4$; telescopes **c)** $1/6$ for $n = 5$; telescopes **e)** -11 for $n = 6$ **7.** $k = 0$ term is 0; index name is irrelevant. **9.** $i = 1$ to $n+1$ **11. a)** 498 **12. a)** -256 **13. a)** 3 for $n = 3$ **14.** 51; 200 **18. a)** $(0, 10)$ **d)** $[0, 1]$ **g)** $(0, 1/10)$ **19. b)** $P_3 \cap P_5 \cap P_7$; true.

Section 0.6

1. $\begin{bmatrix} 6 & 8 \\ 10 & 12 \end{bmatrix}$ **3.** $\begin{bmatrix} 4 & 8 \\ 12 & 16 \end{bmatrix}$ **6. a)** [5 1 9] **d)** 20 **7. a)** $A = \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix} B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$ **d)** $\begin{bmatrix} 20 & 40 & 60 \\ 26 & 52 & 78 \\ 32 & 64 & 96 \end{bmatrix}$ **8. a)** 81

9. a) $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ **11.** $\begin{bmatrix} 285 \\ 335 \\ 560 \end{bmatrix}$ **12.** $n \times n$ matrix **13.** Only A and B **16.** $M^3 = \begin{bmatrix} 1 & 3 & 6 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$ M^n is like M^3 except that $(1,2)$ and $(2,3)$ elements are n and $(1,3)$ element is $n(n+1)/2$. **17.** $c = 22a_1 + 28a_2$. **19.** Columns of A = Rows of B, Columns of B = Rows of C; double sum over r and s of $a_{ir}b_{rs}c_{sj}$ to get (i,j) element of product. **20.** Generalize solution to [19]. **21.** Can compute all powers of any square matrix, no power of any non-square matrix.

Section 0.7

3. a) If n is even, then $2n$ is even. **c)** If there is a 29 Feb. this year, then this is a leap year. **4. a)** False **c)** True **e)** False **5. a)** If $ABCD$ is a parallelogram, then $AB \parallel CD$. **d)** If flowers wilt, then it's hot. **6. b)** If a computer program doesn't terminate, then it's not correct. **e)** If it's not cold, then it won't snow. **7. b)** If n is even, then $2n$ is odd. **9.** (a), (c) and (d) are equivalent; (b) and (e) are equivalent. **11. a)** True **d)** False **g)** True (but it would be false if Π were replaced by x). **13.** Pairs which are negations of each other are $a-d$, $b-c$, $c-e$, $d-f$. **14. a)** Same meaning (but not very good English!) **c,e)** Not good English **16. a)** Vacuously true since there is no k such that $1 \le k \le 0$. **e)** Not vacuously true and neither trivial nor obvious **h)** Not vacuously true but trivial

Supplementary Problems: Chapter 0

1. b) False since x could be -2 **e)** True. **2. b)** 44 **3. b)** 2145 **4. b)** The product of the two integers **c)** Show that if p^i is a factor of m and p^j is a factor of n (for p prime), then one of these is a factor of the gcd and the other is a factor of the lcm. **5. c)** Add $(-1)^k$ under summation. **6. c)** Substitute b/d into $f(x)$, then multiply by d^{n-1} to get result for d, multiply by d^n/b to get result for b. **d)** $1/2, 1, 3/2, 2, 3, 6$ and their negations **8. a)** Number of variations in sign excluding zeros of a_0, $-a_1$, a_2, ..., $(-1)^n a_n$ **9. b)** 0 positive zeros, up to 10 negative zeros **10.** Sum from $i = 1$ to n of $(in!)/[(n-i)!i!]$.